What The Quantum Control Can Do For Us?

Tzyh-Jong Tarn

Center for Quantum Information Science and Technology

Tsinghua University, P.R. China

Department of Electrical and Systems Engineering

Washington University, St. Louis, USA

Goals

- To extend the feedback design method to quantum domain
- To apply the result to very important quantum control problems
- To demonstrate the effectiveness of the obtained results
- To draw your attention to this new growing and challenging field

Contents

Early-Stage History

- **1980s Tarn Group (US)**

Modeling, controllability and invertibility, non-demonition filtering

- **1980s Belavkin (Britain)**

Filtering and stochastic control

- **Late 1980s –1990s Rabitz Group (US)**

Optimal control theory and Learning control in ultrafast chemical control systems

- **Late 1980s –1990s Tannor, Kosloff and Rice (US)**

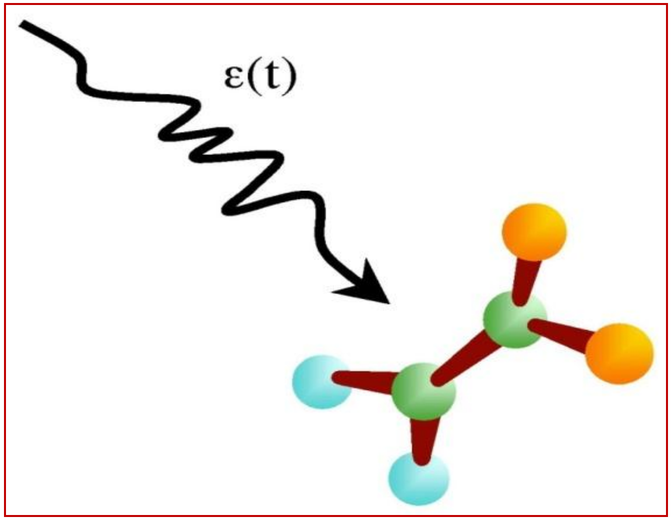
Bond-selective control of chemical reactions

Opportunities for Quantum Control

- Extremely high-precision measurement
- Ultracold systems in condensed matter science
- High-intensity and short-wavelength light sources
- Ultrafast control on the motion of atoms/electrons
- Quantum engineering on the nanoscale structures
- Quantum computation, data security and encryption

From “Controlling the quantum world: the science of atoms, molecules and photons”, Physics 2010, National Research Council (2007)

Laser Control of Molecules



Successful experiments have been reported to date over 150 systems in over 20 physical and chemical categories

Research Groups

- *H. Rabitz* (Princeton)
- *T. Weinacht* (Stonybrook)
- *R. Levis* (Temple)
- *G. Gerber* (Würzberg)
- *M. Zanni* (Wisconsin)
- *L. Woeste* (Berlin)
- *T. Brixner* (Würzberg)
- …..

Physical Implementations of Quantum Computer

- **Nuclear Magnetic Resonance (NMR)**
- **Superconducting Josephson Junction**
- **Quantum Tunneling**
- **Ion Trap**
- **Quantum Dot**
- **Cavity QED**

Nobel Prize in Physics, 2012

In the recent Nobel Prize awards 2012, **Serge Haroche** and **David Wineland** were jointly awarded the Nobel Prize for Physics for their *"ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems"*.

Classical Mechanics

Determine the position of a particle at any time,

$$x(t) \Rightarrow \boldsymbol{v}(\boldsymbol{t}) = \frac{\boldsymbol{dx}}{\boldsymbol{dt}} \Rightarrow \boldsymbol{p}(\boldsymbol{t}) = \boldsymbol{mv}(\boldsymbol{t}) \Rightarrow \boldsymbol{T} = \frac{1}{2}\boldsymbol{mv}^2(\boldsymbol{t})$$

To determine $x(t)$, using

$$\boldsymbol{F} = \boldsymbol{ma}$$

For conservative system,

$$\boldsymbol{F} = -\frac{\partial \boldsymbol{V}}{\partial \boldsymbol{x}} \Rightarrow \boldsymbol{m}\frac{\boldsymbol{d}^2\boldsymbol{x}}{\boldsymbol{dt}^2} = -\frac{\partial \boldsymbol{V}}{\partial \boldsymbol{x}}$$

with initial conditions

Elements of Quantum Mechanics

A quantum particle is described by its wavefunction

$\psi(t, x)$ determined by the Schrödinger equation

$$i\hbar \frac{\partial \left|\psi(t,x)\right\rangle}{\partial t} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) \right] \left|\psi(t,x)\right\rangle$$

where the ket $\left|\psi(t,x)\right\rangle$ represents the quantum state

$\left|\psi(t,x)\right|^2 \mathrm{d}x =$ Probability of finding the particle between x and $x + \mathrm{d}x$ at time t

Evolution is unitary to preserve the probability

Non-Classical Features

Heisenberg Uncertainty Principle

Two non-commuting observables can not be measured simultaneously accurately.

For example: $[\hat{x}, \hat{p}] = i\hbar \Rightarrow \Delta x \cdot \Delta p \geq \dfrac{\hbar}{2}$

where $[.,.]$ is the commutator

NO classical trajectory can be defined for a quantum particle !

Non-Classical Features

Entanglement

tensor product + superposition

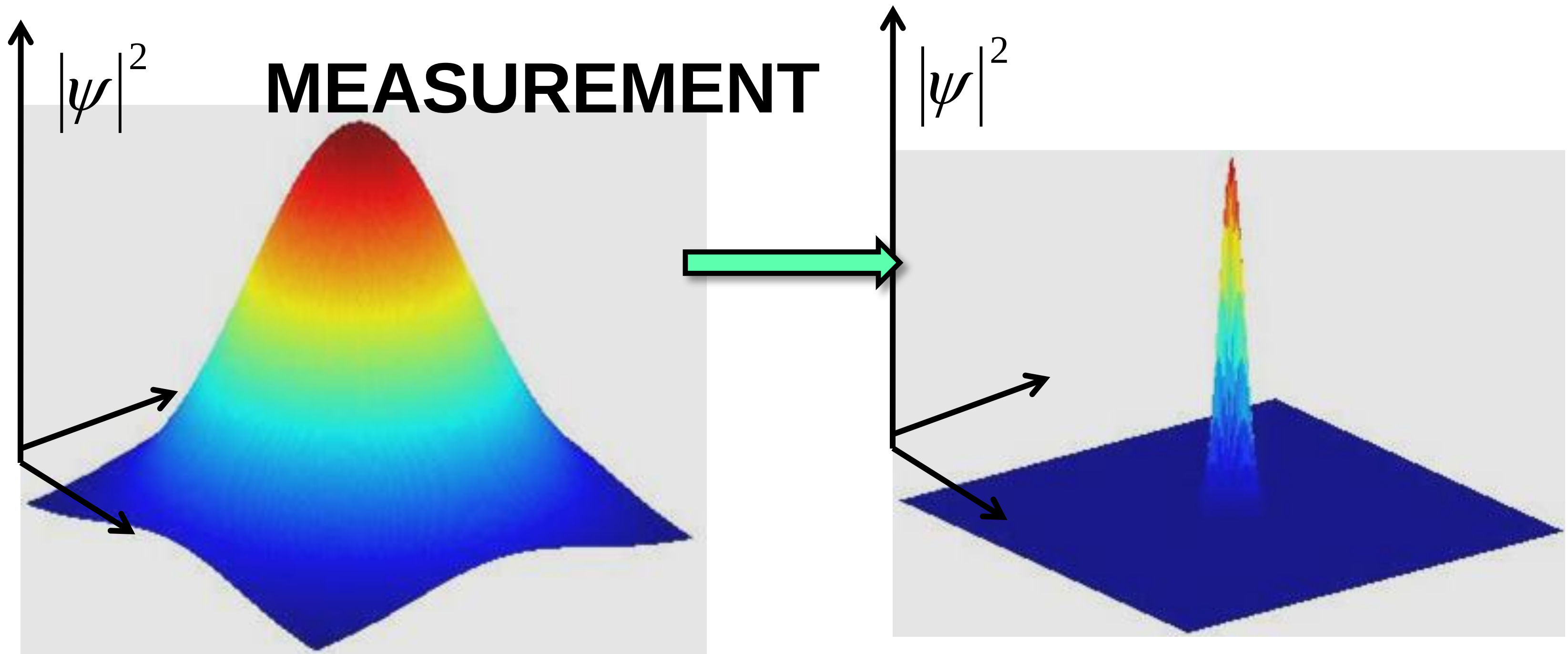
System of n qubits $\quad \boldsymbol{H} = \underbrace{\mathbf{C}^2 \otimes \cdots \otimes \mathbf{C}^2}_{\boldsymbol{n}}$ (**dim = 2*ⁿ***)

Entangle 250 atoms together and one can simultaneously encode more numbers than there are atoms in the known universe!

Non-Classical Features

Quantum Measurement

Wavefunction collapses instantaneously and randomly after measurement



Continuous Measurements

Significant progress in the late 1970s and 80s shows that, within the axiomatic framework of Quantum Mechanics, a consistent formulation of measurement theory was possible based on Positive Operator Value Measure (POVM) in continuous time.

Applications to

- Photon Detection
- Currently applied to Quantum Dots

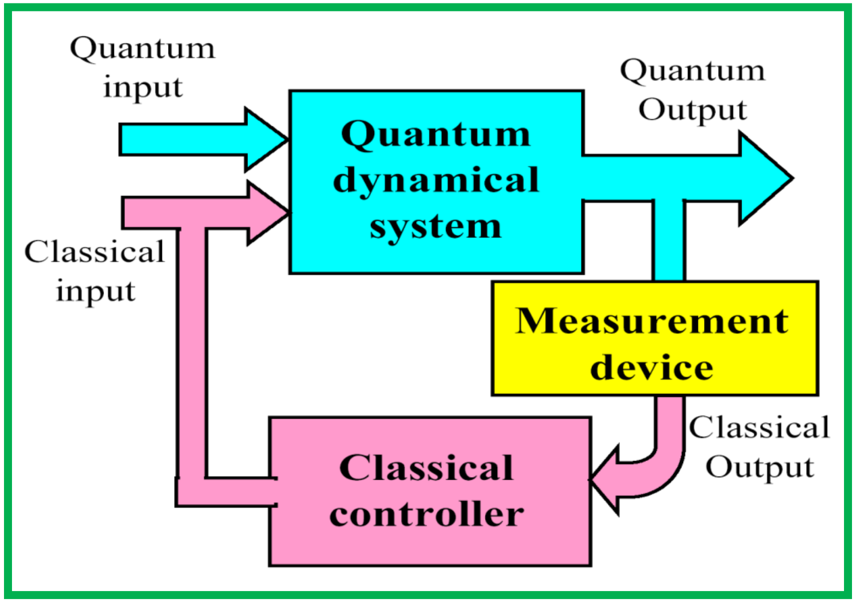
Quantum feedback schemes

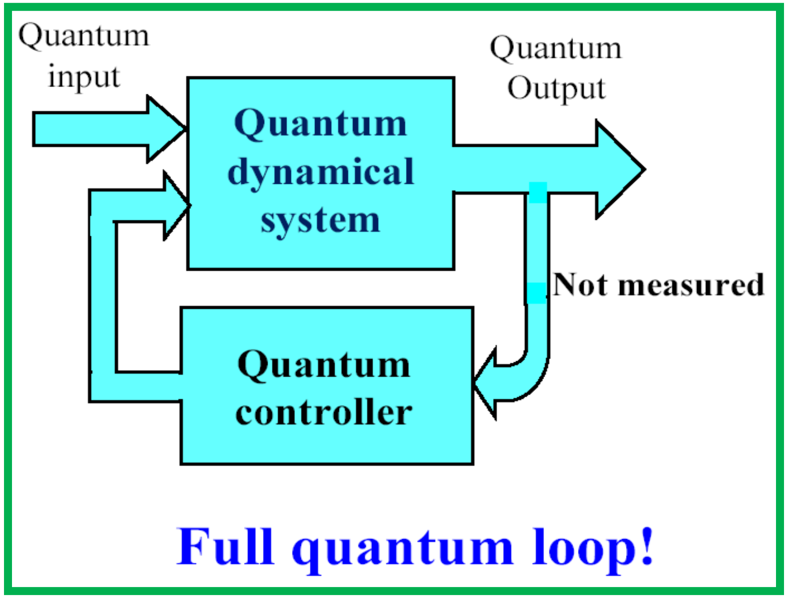
Measurement-based feedback

(Classical control)

Coherent feedback

(Quantum control)





Essential distinction: the control loop is quantum or not !

Given a quantum control system:

$$\dot{x} = f(x) + \sum_i g_i(x)u_i(x)$$

1.Measurement-based feedback (classical Control)

$$\dot{x} = f(x) + \sum_i g_i(x)u_i(x); \quad y = h(x)$$

design $u_i(x) = K(h(x))$

2.Coherent feedback (Quantum Control)

$$\dot{x} = f(x) + \sum_i g_i(x)u_i(x) + \sum_j l_j(x)v_j(x)$$

adding $\sum_j l_j(x)v_j(x)$, then we could design $u_i(x)$,$v_j(x)$

The First Problem: Motivations

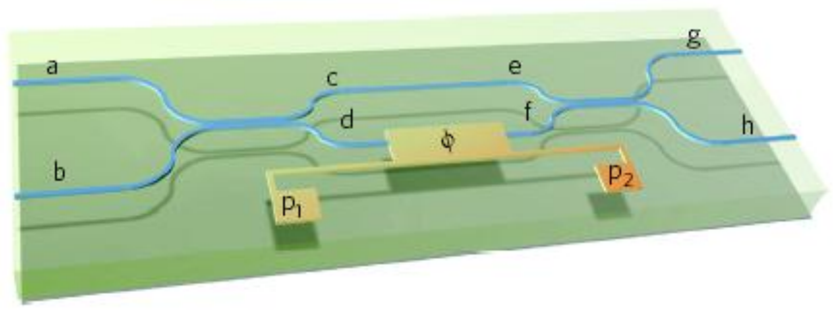
A long-standing question in quantum control

Is there any problem that can be accomplished by quantum control, but not by classical control?

Nonlinear quantum optics on chip

Natural nonlinearity is too weak to demonstrate novel quantum optical phenomena.

Is there any way to artificially generate and enhance the desired nonlinearity?



Science **325**, 1221 (2009)

Nature Photonics **3**, 346 (2009)

The aims of our work

To establish a paradigm for the first question

Classical feedback cannot generate quantum nonlinearity

(see our work in Physical Review A **82**, 022101 (2010));

However, full quantum feedback can do It !

The second question will be answered in the examples shown later

Previous work in coherent feedback

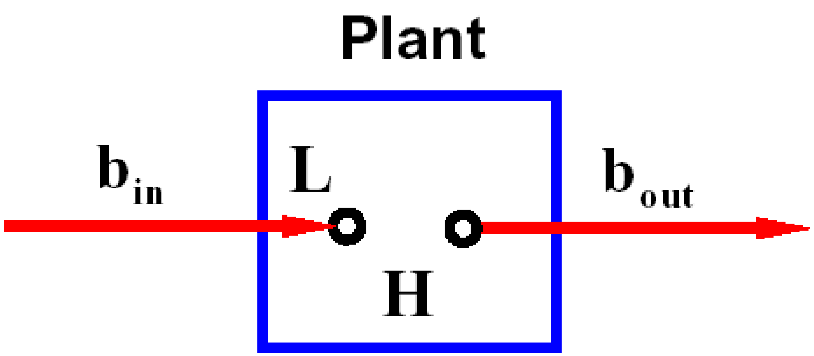
General theory of linear coherent feedback

Hudson-Parthasarathy (HP) model: IEEE TAC **54**, 2530 (2009)

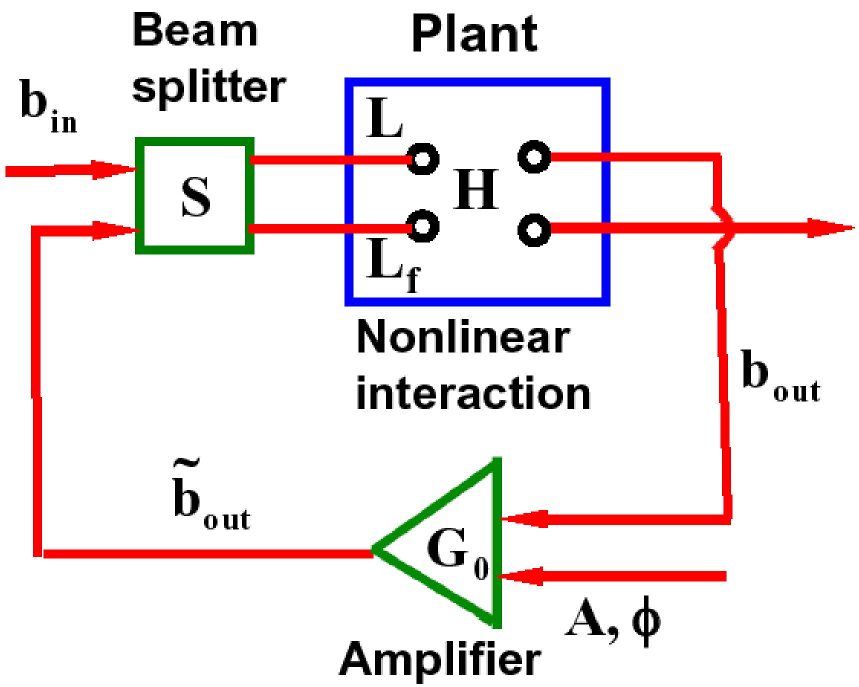
Quantum transfer function: IEEE TAC **48**, 2107 (2003),
Phys. Rev. A **81**, 023804 (2010).

Existing methods are not applicable to nonlinear coherent feedback !

The proposed feedback system



Open-loop system



Closed-loop system

Our main result

Uncontrolled system dynamics

$$\dot{\rho} = -i[H,\rho] + L\rho L^\dagger - L^\dagger L\rho/2 - \rho L^\dagger L/2$$

Controlled system dynamics

$$\dot{\rho} = \underbrace{-i[H_{\text{eff}},\rho]}_{\text{Feedback-induced nonlinear Hamiltonian}} + \underbrace{\tilde{L}\rho\tilde{L}^\dagger - \tilde{L}^\dagger\tilde{L}\rho/2 - \rho\tilde{L}^\dagger\tilde{L}/2}_{\text{Decoherence induced by the input vacuum field}}$$

$$+\underbrace{(N+1)D[\tilde{L}]\rho + ND\left[\tilde{L}^\dagger\right]\rho + \left[M^*\left(\tilde{L}\rho\tilde{L} - \tilde{L}^2\rho/2 - \rho\tilde{L}^2/2\right) + \text{h.c.}\right]}_{\text{Decoherence induced by the quantum amplifier}}$$

$$H_{\text{eff}} = \underbrace{H}_{\text{Uncontrolled Hamiltonian}} + \underbrace{\frac{i}{2}\sqrt{G_0}}_{\text{Ehanced by quantum amplifier}} \underbrace{\left[\left(L+L^\dagger\right)S^\dagger L_f - L_f^\dagger S\left(L+L^\dagger\right)\right]}_{\text{Control induced nonlinear term}}$$

$$+\underbrace{\sqrt{G_0}A\left[(\cos\phi)L + i(\sin\phi)S^\dagger L_f + \text{h.c.}\right]}_{\text{This term is introduced to cancel the linear term}}$$

Main difficulties for analysis

- Non-commutative quantum probability theory
- Analysis of quantum nonlinearity
- Noise is defined on the infinite-dimentional quantum Fock space
- Hard to obtain a quantum closed-form equation

Key Derivations (I)

1. Obtain the quantum stochastic diffirential equation (QSDE) of the total system including the input field and the amplifier

$$d\bar{U} = \left\{-i\bar{H}dt - \tfrac{1}{2}\bar{L}^\dagger\bar{L}dt + dB^\dagger\bar{L} - \bar{L}^\dagger dB + \mathrm{tr}[(\bar{S} - I)d\Lambda]\right\}\bar{U}$$

$$\bar{S} = S^2, \bar{L} = L_f + S(\sqrt{\kappa}c + L), \bar{H} = H + H_c + \tfrac{i}{2}\sqrt{\kappa}\left(L^\dagger c - c^\dagger L\right)$$
$$+\tfrac{i}{2}\left[\left(L^\dagger + \sqrt{\kappa}c^\dagger\right)S^\dagger L_f - L_f^\dagger S(L + \sqrt{\kappa}c)\right]$$

Classical Ito rule

$$dWdW^* = dW^*dW = dt$$

Quantum Ito rule

$$dBdB^\dagger = dt, \qquad dB^\dagger dB = 0,$$
$$dBd\Lambda = dB, \qquad d\Lambda dB = 0,$$
$$dB^\dagger d\Lambda = 0, \qquad d\Lambda dB^\dagger = dB^\dagger,$$
$$d\Lambda d\Lambda = d\Lambda$$

Key Derivations (II)

2. Using singular perturbation method to adiabatically eliminate the degrees of freedom of the quantum amplifier

Decomposition $\bar{U} = UV_\varepsilon$

$$dU = -i\left\{H + \tfrac{i}{2}L^\dagger S^\dagger L_f + \left[-i\left(L^\dagger - L_f^\dagger S\right) - 2Ae^{i\phi}\right]a_\varepsilon + \mathrm{h.c.}\right\}U\,dt$$

Quantum Ornstein-Uhlenbeck noise

$$a_\varepsilon = \sqrt{\tfrac{\varepsilon}{\kappa_0}}\left[G_1\left(c + c^\dagger\right) + G_2\left(c - c^\dagger\right)\right] + \tfrac{\kappa_0}{\kappa_0 - \xi_0}\int_0^t G_1(t-\tau)\left[S\,dB_\tau + \tfrac{1}{2}\left(L + S^\dagger L_f\right)d\tau\right] + \tfrac{\kappa_0}{\kappa_0 + \xi_0}\int_0^t G_2(t-\tau)\left[\tfrac{1}{2}\left(L + S^\dagger L_f\right)d\tau + S\,dB_\tau\right]$$

$$G_{1,2}(\tau) = \frac{\kappa_0 \pm \xi_0}{4\varepsilon}\exp\left[-\frac{\kappa_0 \pm \xi_0|\tau|}{2\varepsilon}\right] \to \delta(\tau), \qquad \varepsilon \to 0$$

Key Derivations (III)

3. Average out all the noises to obtain the master equation

$$\dot{\rho} = -i[H_{\mathrm{eff}}, \rho] + D[\tilde{L}]\rho + (N+1)D[\tilde{L}]\rho + ND\left[\tilde{L}^{\dagger}\right] + M^{*}\left(\tilde{L}\rho\tilde{L} - \tilde{L}^{2}\rho/2 - \rho\tilde{L}^{2}/2\right) + \mathrm{h.c.}$$

$$\rho = U\rho_0 U^{\dagger};\ D[\tilde{L}]\rho = \tilde{L}\rho\tilde{L}^{\dagger} - \tilde{L}^{\dagger}\tilde{L}\rho/2 - \rho\tilde{L}^{\dagger}\tilde{L}/2$$

$$\tilde{L} = L - S^{\dagger}L_f;\ N = G_0 - 1;\ \ M = \sqrt{(G_0 - 1)G_0}$$

Extended quantum Ito rule

$$d\tilde{B}d\tilde{B}^{\dagger} = (N+1)dt, d\tilde{B}^{\dagger}d\tilde{B} = Ndt, d\tilde{B}d\tilde{B} = Mdt$$

Cannot be obtained from HP model or quantum transfer function

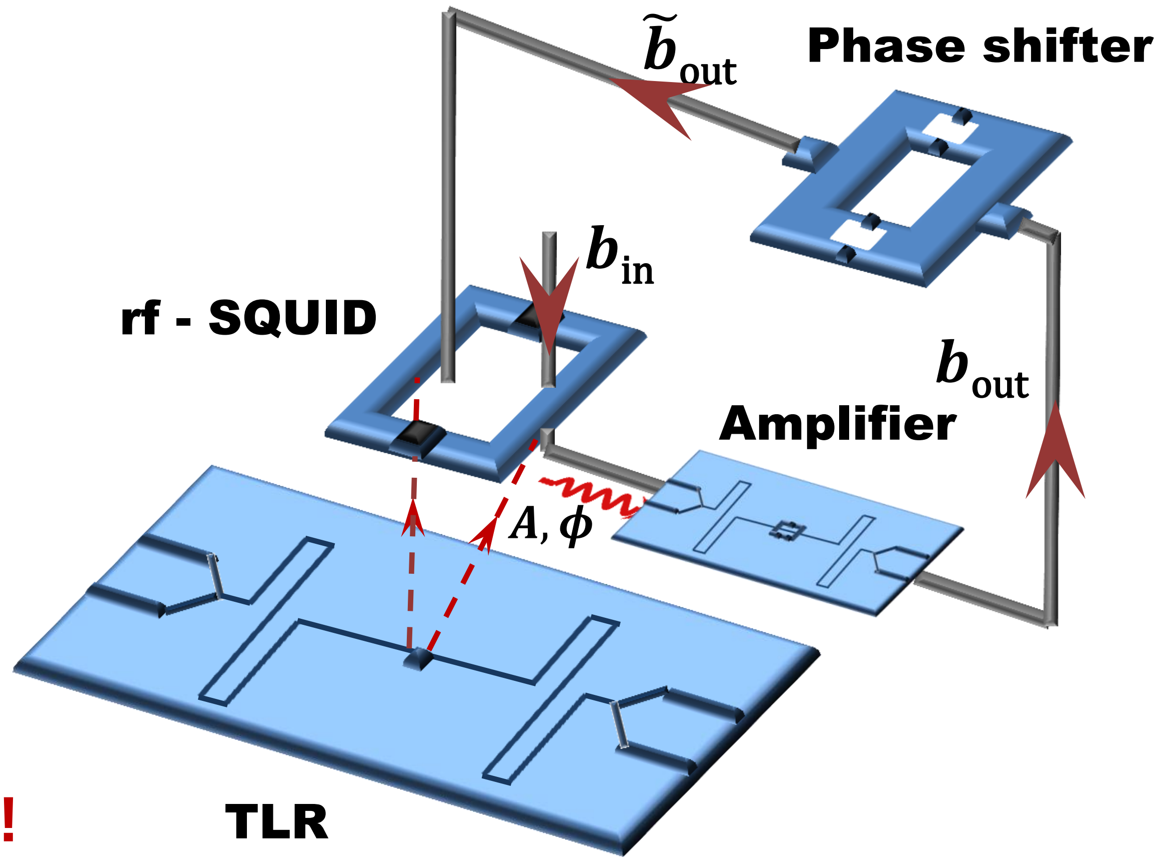
] Detailed derivations can be found in IEEE Trans. Automat. Contr. **57**, 1997 (2012).

Is it possible for experimental implementation ?

Generation of Kerr effect

$$H_{\text{eff}} = \underbrace{\chi\left(a^{\dagger}a\right)^{2}}_{\text{Induced Kerr term}} + (\omega_a - \delta)a^{\dagger}a$$

$$\delta = 2A\sqrt{G_0\gamma_a},$$
$$\chi = 2\sqrt{G_0\gamma_a}$$



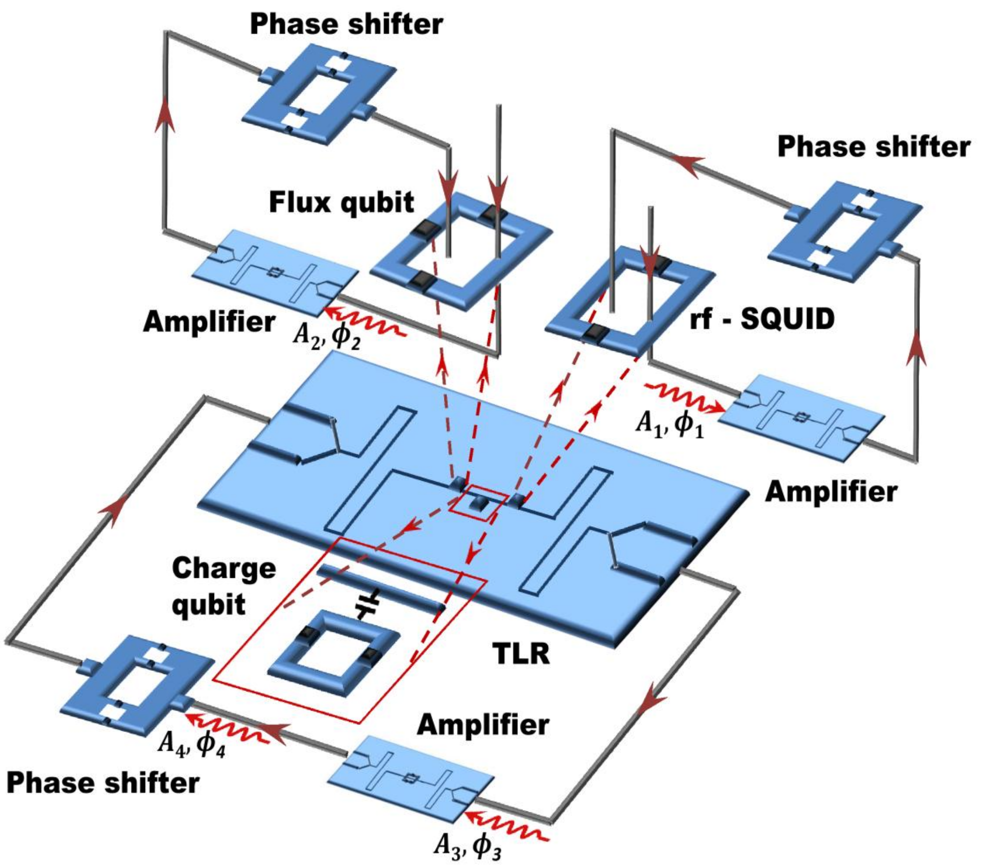
χ is 10^4-10^5 stronger than its natural value!

Potential applications: superposed quantum state preparation, quantum non-demolition measurement…

Synthesis of high-order Hamiltonian

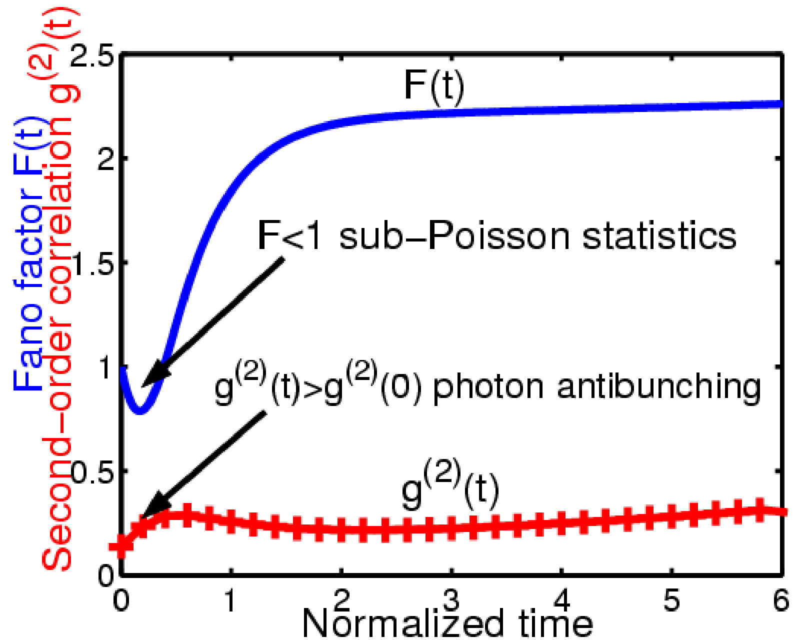
$$H_{\text{eff}} = \omega_a a^\dagger a + \underbrace{\sum_{k=1}^{4} \chi_k \left[\frac{a^\dagger + a}{\sqrt{2}}\right]^k}_{\text{Induced nonlinear terms}}$$

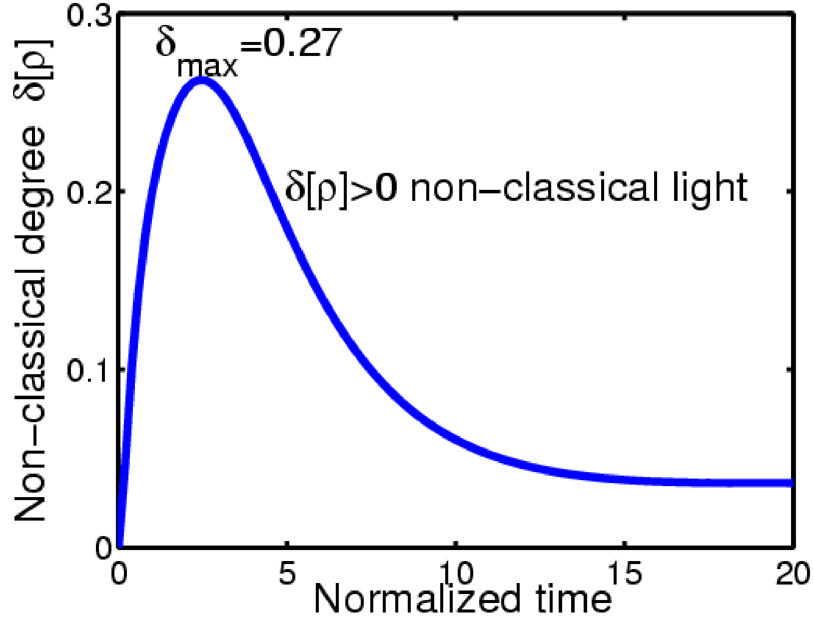
$$\chi_1 = A_4\sqrt{2\gamma},$$
$$\chi_2 = 4A_1\sqrt{G_1\gamma_1} - 2A_3\sqrt{G_3\gamma_3},$$
$$\chi_3 = 2\sqrt{G_3\gamma\gamma_3},$$
$$\chi_4 = 2\sqrt{G_1\gamma\gamma_1}$$



Applications: nonlinear quantum optics and quantum information

Construction of non-classical light

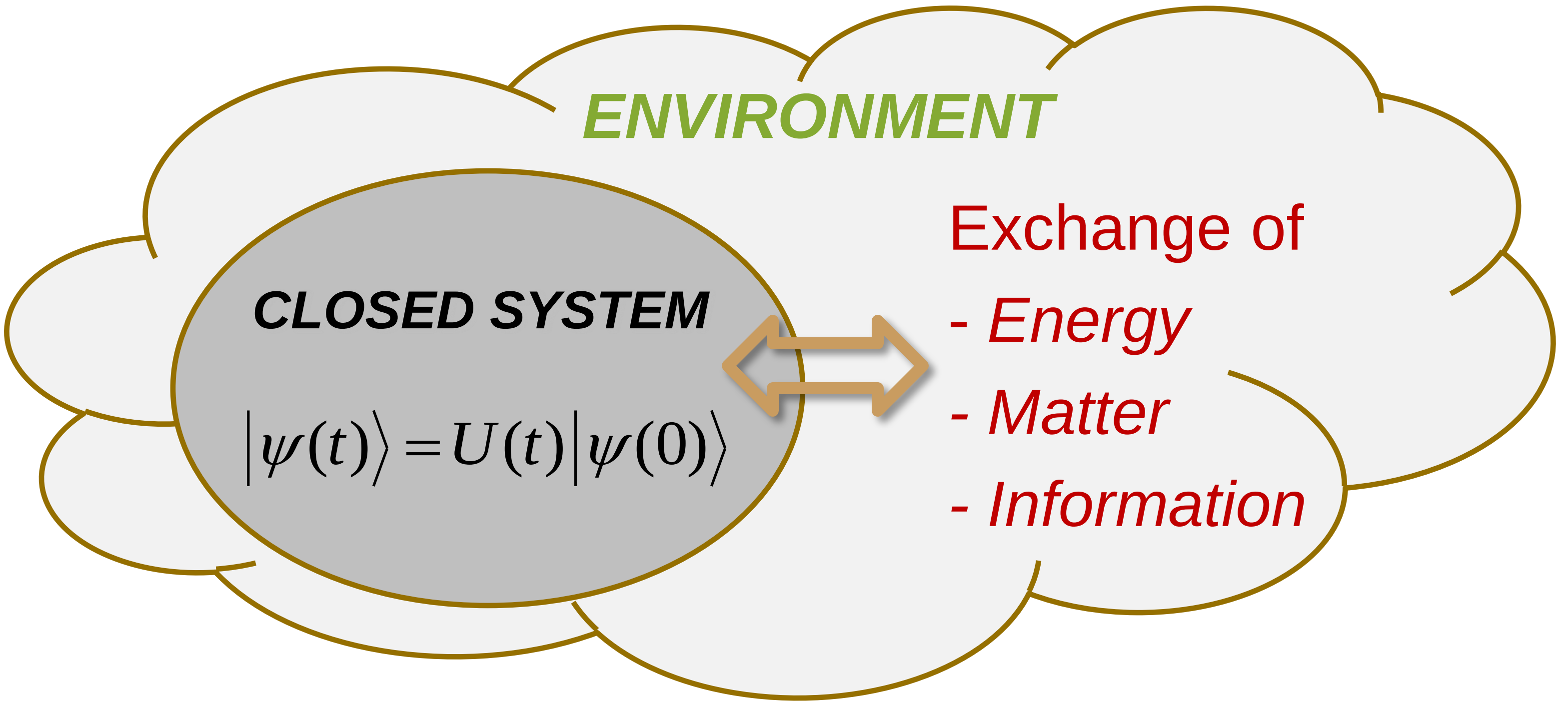




1) **Highly non-classical light can be generated.**

2) **Sub-Poisson statistics and photon antibunching can be observed;**

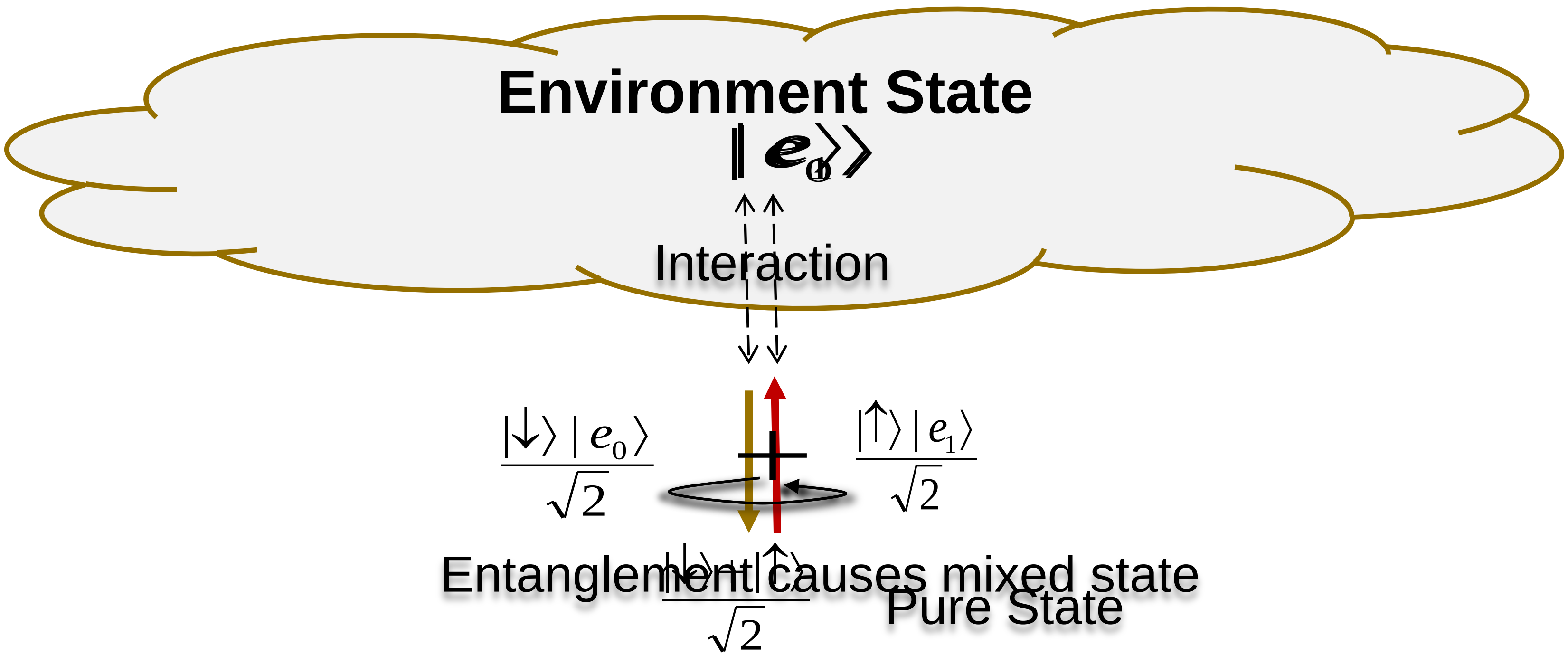
Open Quantum System



Unitary evolution of total system (Closed)

$$\hat{H} = \hat{H}_{SYSTEM} + \hat{H}_{ENVIRONMENT} + \hat{H}_{INTERACTION}$$

What is Decoherence ?



- Loss of quantum superposition
- Extremely quick for macroscopic systems
- Quantum analog of damping / friction

Control System Model

Composite-system Schrödinger Equation

$$i\hbar\frac{\partial\left|\Psi(t)\right\rangle}{\partial t}=[\hat{H}_S(t)\otimes I_E+I_S\otimes\hat{H}_E(t)+\hat{H}_{SE}(t)+\sum_{i=1}^{r}u_i(t)\hat{H}_i\otimes I_E]\left|\Psi(t)\right\rangle$$

Free Hamiltonian of the system

Hamiltonian for the environment

Interaction Hamiltonian between the system and the environment

controls

State of the Total System

Semiclassical Control Hamiltonians

Decoherence is NOT Classical Noise

Consider the classical system

classical noise

$$\dot{x} = f(x) + \sum_i u_i(t) g_i(x) + w(t) p(x)$$

- Dimensionality of the system remains the same with the ADDTION of classical noise.
- Dimensionality of the quantum system changes from dim H_S to dim H_S x dim H_E
- Leads to entanglement and loss of information

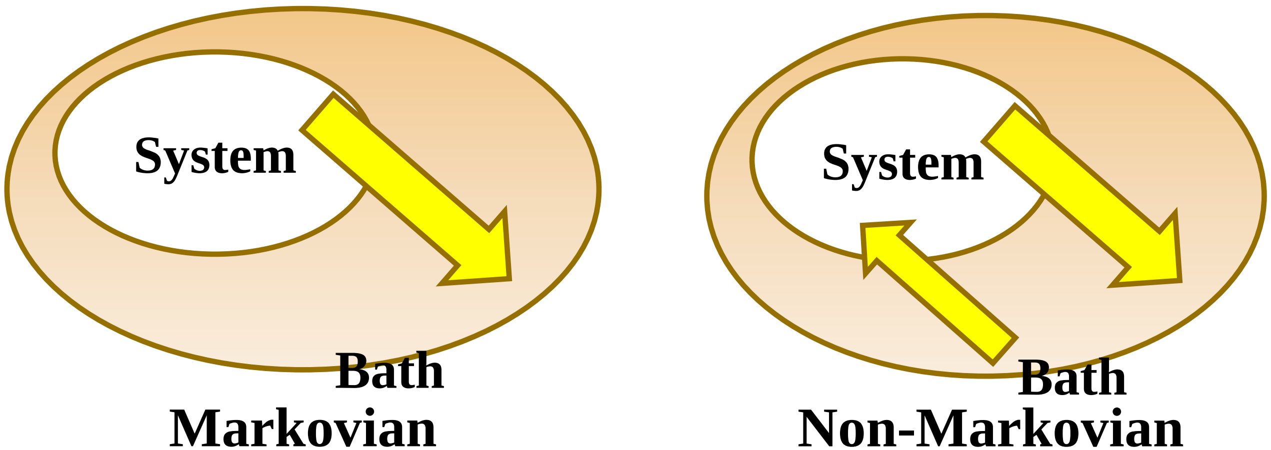
The Second Problem: Disturbance Decoupling

Coherent Quantum Feedback Rejection of Non-Markovian Noises

Motivations

- Why quantum information revolution has not come?

  Decoherence



- How to utilize coherent feedback scheme to suppress non-Markovian decoherence ?

Objectives

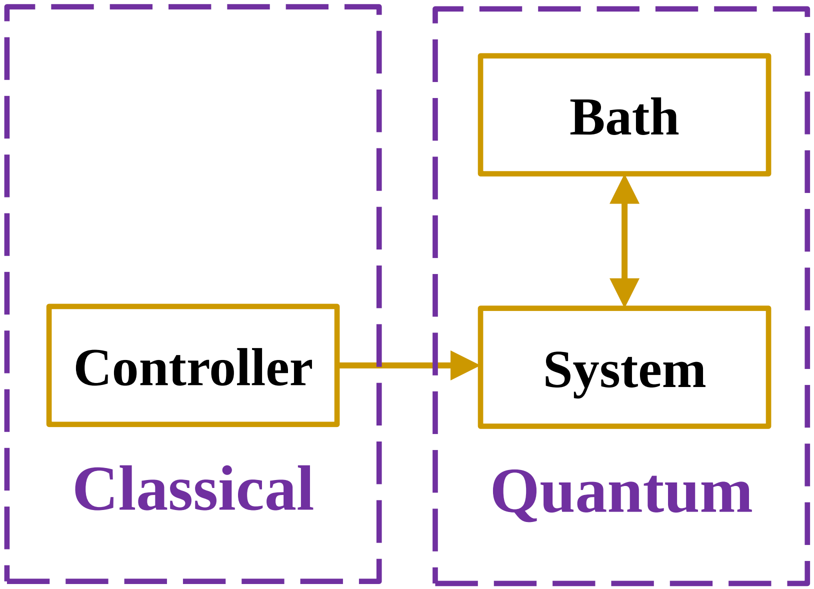
- **Model Non-Markovian Open Quantum Systems for Coherent Feedback;**
- **Investigate Mechanism of Non-Markovian Noises Rejection via Coherent Feedback.**

Backgrounds (I)

NM Decoherence Control

- **Dynamical Decoupling Approach**
- **Optimization Method**

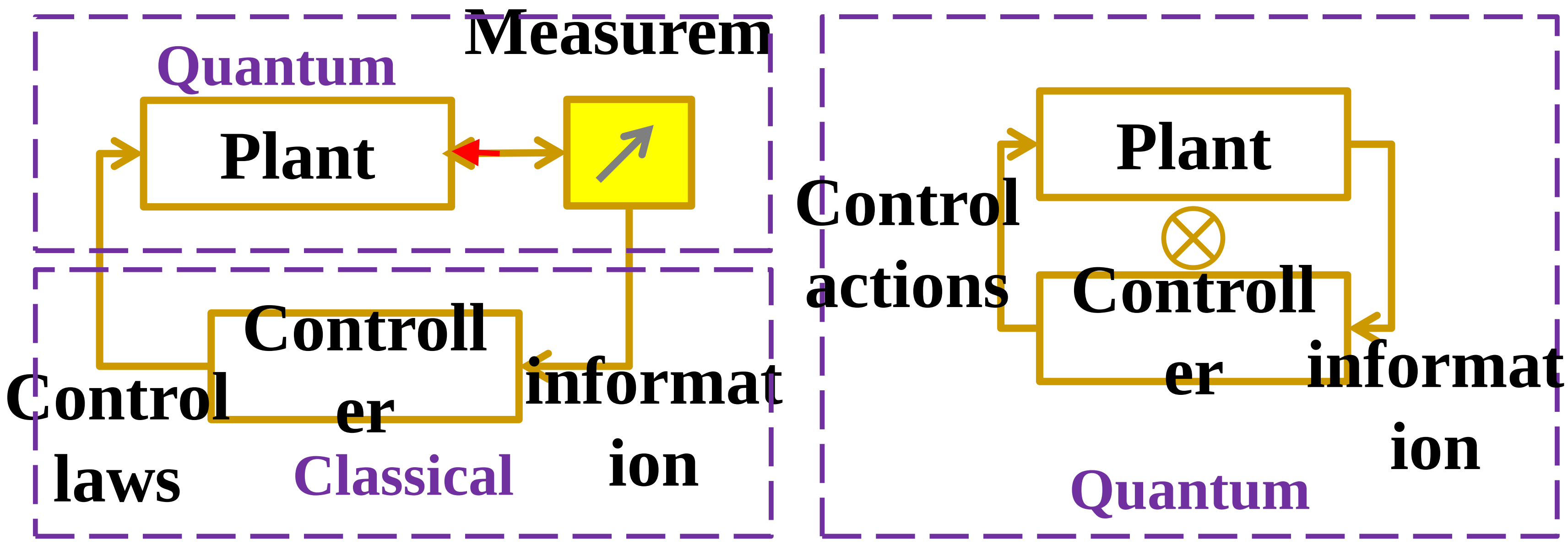
Disadvantage:
high energy cost;
heavy computation burden.



Open Loop Control

Backgrounds (II)

Quantum Feedback Control (QFC)

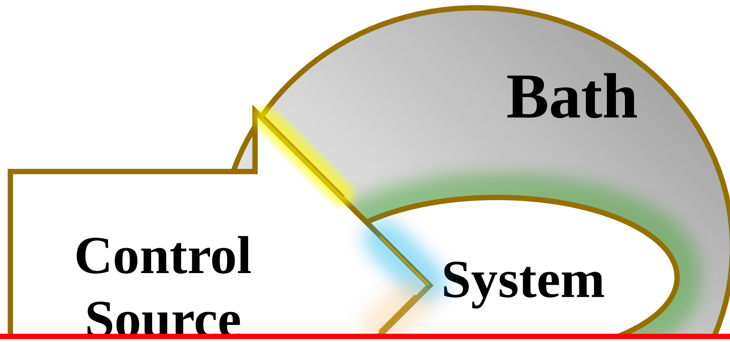


Measurement-based QFC

Coherent QFC

Existing models are not applicable to non-Markovian open quantum systems!

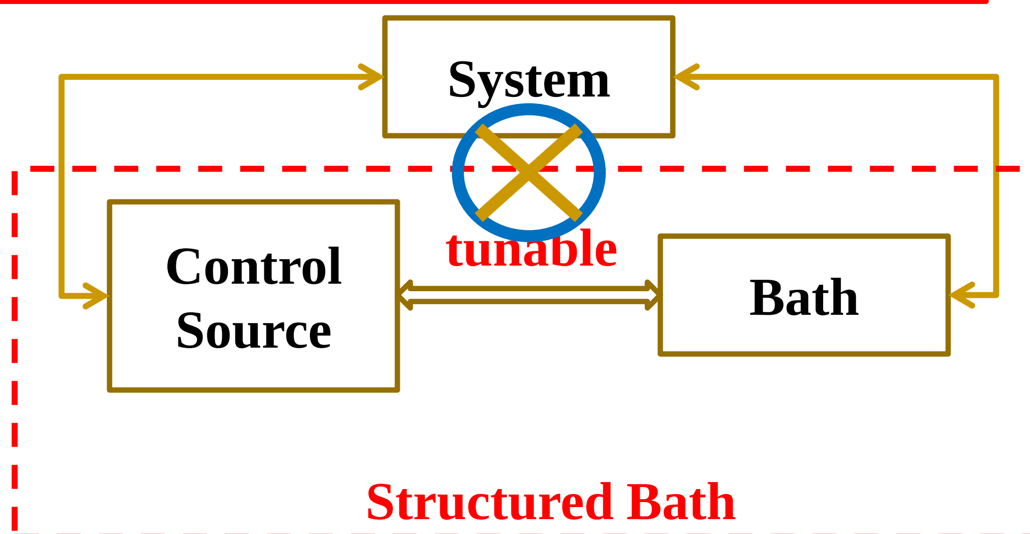
Our Scheme



The introduced control source not only interacts with the system but also … ath.

Direct Coherent Feedback

The noise spectrum of structured bath is promising to be modulated.



Internal Model Principle

In Summary

Every good regulator incorporates a model of the outside world!

Main Difficulties

Infinite Dimensional System

Integral-differential Equation (Memory Effect)

Colored Noises

Phys. Rev. A 86, 052304 (2012).

http://link.aps.org/doi/10.1103/PhysRevA.86.052304.

Modeling of Coherent Feedback Loop

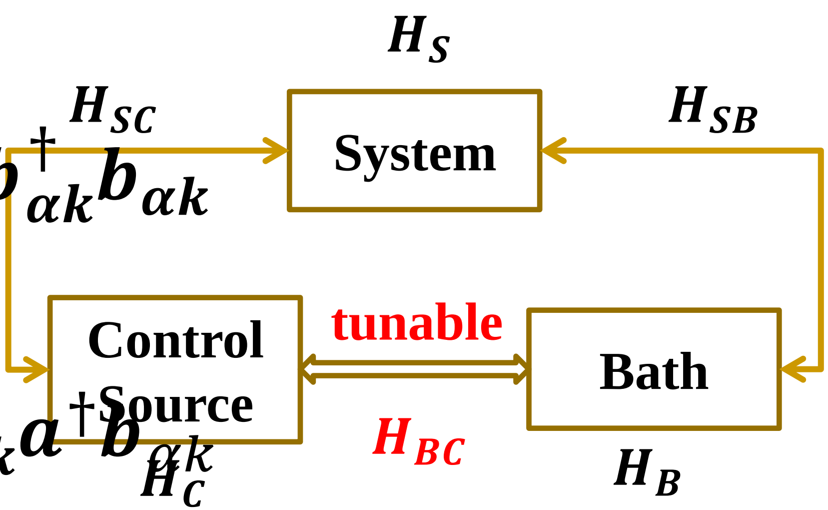
- **Total Hamiltonian and Diagram**

$$H = H_S + H_B + H_C + H_{SB} + H_{SC} + H_{BC}$$

where $H_S = \omega_a a^{\dagger} a$

$$H_B + H_C = \sum_{\alpha=B,C} \sum_k \omega_{\alpha k} b^{\dagger}_{\alpha k} b_{\alpha k}$$

$$H_{SB} + H_{SC} = \sum_{\alpha=B,C} \sum_k V_{\alpha k} a b^{\dagger}_{\alpha k} + V^{*}_{\alpha k} a^{\dagger} b_{\alpha k}$$



$$H_{BC} = \sum_k \sum_{k'} F_{kk'} b_{Bk} b^{\dagger}_{Ck'} + F^{*}_{kk'} b^{\dagger}_{Bk} b_{Ck'}$$

Resonance assumption $F_{kk'} = f_k \cdot \delta(k, k'),$ $\quad f_k = re^{i\theta}$

Modeling of Coherent Feedback Loop

- **NM Quantum Langevin Equation:**

$$\dot{a}(t) = -\mathrm{i}\omega_a \underbrace{a(t)}_{\text{System Operator}} - \int_0^t \mathrm{d}\tau \underbrace{G(t-\tau)}_{\text{Memory Kernel Function}} a(\tau) - \mathrm{i} \underbrace{b_n(t)}_{\text{Equivalent Noise}}$$

which can be solved as

$$a(t) = u(t)a(0) + \int_0^t \mathrm{d}\tau\, u(t-\tau)\, b_n(\tau)$$

where Green function $u(t)$ satisfies

$$\dot{u}(t) = -\mathrm{i}\omega_a u(t) - \int_0^t \mathrm{d}\tau G(t-\tau) u(\tau)\,, u(0) = 1$$

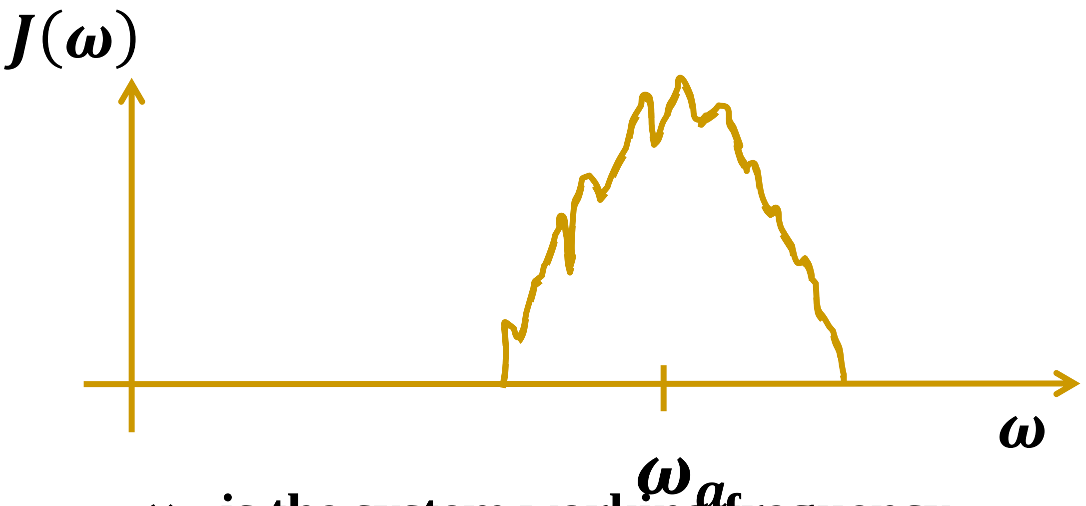
Noise spectrum and control parameters are combined in $G(\tau)$.

NM Decoherence Suppression

- **Memory Kernel Function without Feedback**

$$G(\tau) = \int_{\Omega} \frac{\mathrm{d}\omega}{2\pi} J(\omega) e^{-\mathrm{i}\omega\tau}$$

where $J(\omega) = 2\pi\varrho(\omega)(|V_B(\omega)|^2 + |V_C(\omega)|^2)$



Physically, decoherence is caused by the resonance between the system's working frequency and the noise spectrum.

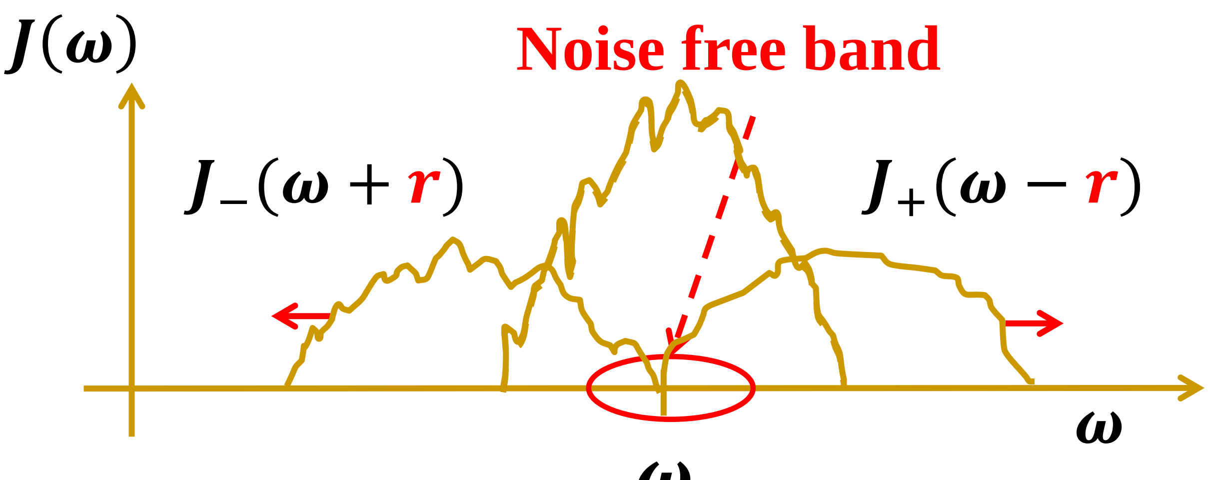
ω_a is the system working frequency

NM Decoherence Suppression

- Memory Kernel Function **with** Feedback

$$G(\tau) = \int_{\omega_L + r}^{\omega_U + r} \frac{d\omega}{2\pi} J_+(\omega - r) e^{-i\omega\tau} + \int_{\omega_L - r}^{\omega_U - r} \frac{d\omega}{2\pi} J_-(\omega + r) e^{-i\omega\tau}$$

where $J_\pm(\omega) = \pi\varrho(\omega)\left|V_B(\omega) \pm V_C(\omega) e^{-i\theta}\right|^2$



ω_a is the system working frequency

The effectiveness of decoherence suppression depends on how close the working frequency is to the nearest edge of the noise free band

Example of Photonic Crystals

Angular Frequency of the Cavity: $\omega_a = 10\text{GHz}$

Central Frequency of the Baths $\omega_\alpha = 10\text{GHz}, \alpha = B, C$

Coupling Strength

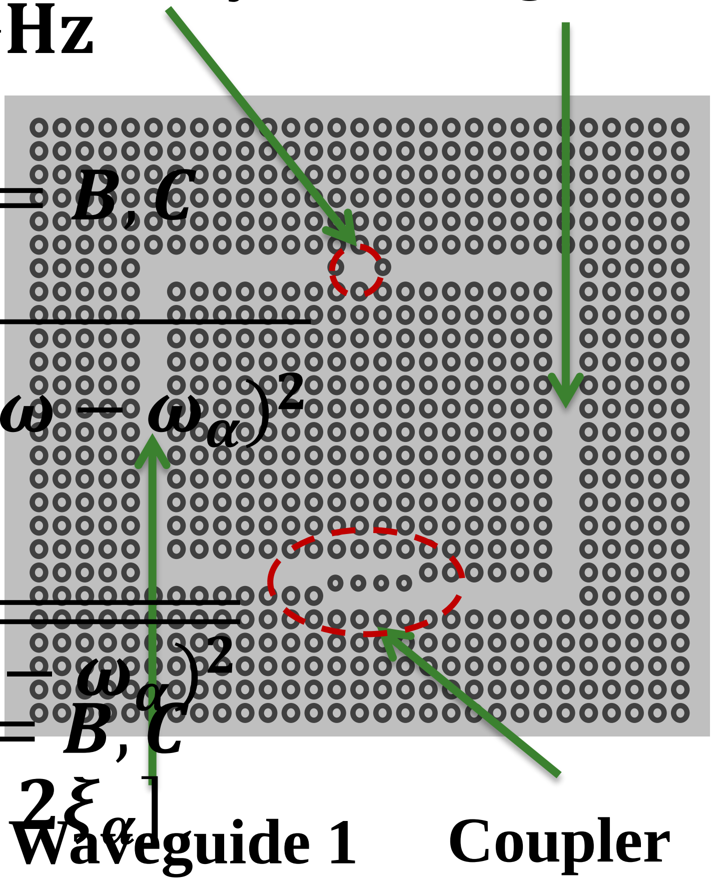
$$V_\alpha(\omega) = \frac{\eta}{\sqrt{2\pi}}\sqrt{4\xi_\alpha^2 - (\omega - \omega_\alpha)^2}$$

Density of the State

$$\varrho_\alpha(\omega) = \frac{1}{\sqrt{4\xi_\alpha^2 - (\omega - \omega_\alpha)^2}}$$

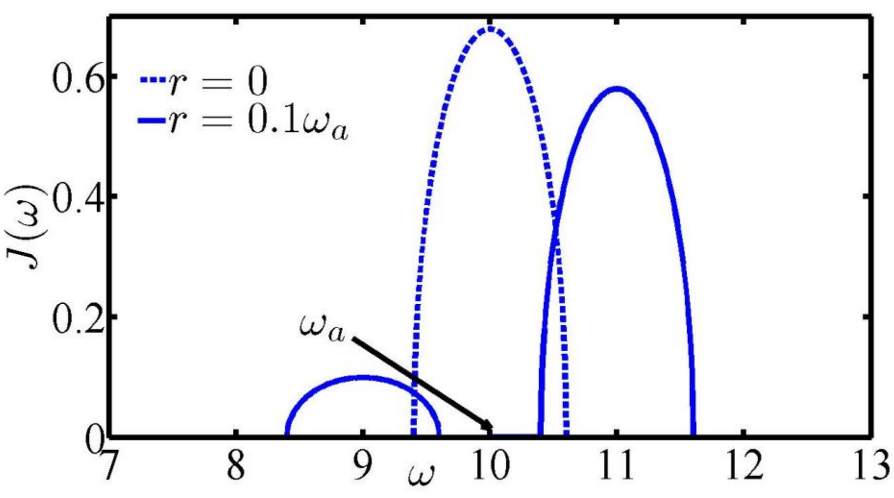
$$\xi_\alpha = 0.3\text{GHz}, \alpha = B, C$$

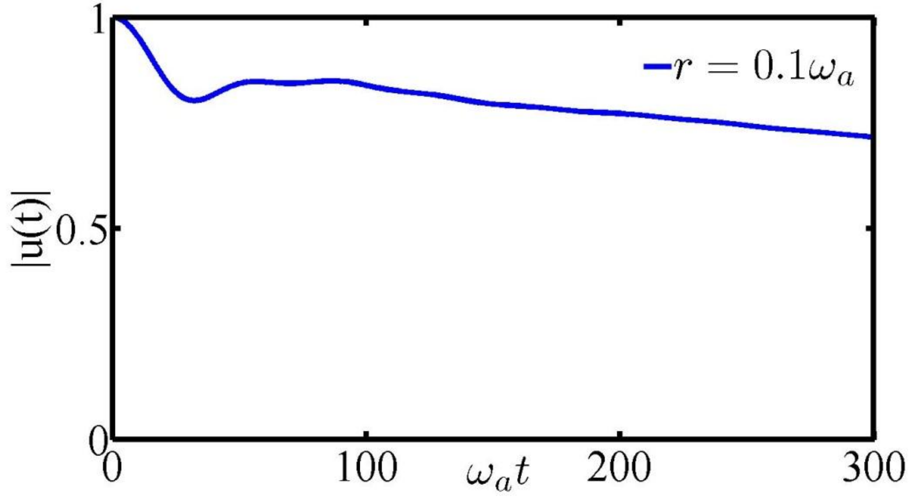
Noise Band $[\omega_\alpha - 2\xi_\alpha, \omega_\alpha + 2\xi_\alpha]$



Example of Photonic Crystals

- **Results of Decoherence Suppression**





The variation of the spectrum and the dynamics of |u(t)| under various feedback parameters *r*, $\eta = 0.3$, θ=$\pi/4$

Conclusions

Main contributions

1) Quantum feedback nonlinearization (QFN): different from the traditional feedback linearization in classical control !

2) A new non-Markovian coherent feedback model (no one discussed before)

3) QFN can achieve more than the traditional physical design.

Open up new dimensions of research

1) Systematic design of coherent feedback control;

2) Frequency domain synthesis and design methods for quantum control;

3) Applications to quantum optics and quantum information.

4) Applications to solid-state-based quantum information processing.

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Dr. *Narayan Ganesan*

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Thank you!